Using psychophysics to ask if the brain samples or maximizes:

Appendix

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1 The influence of a uniform prior distribution on the just-noticeable difference

In this article we analyze an experiment where stimuli are sampled from a Gaussian distribution. It is customary in certain psychophysical experiments to sample such stimuli from a uniform distribution. Therefore we examine the effect of a uniform prior on measured JND for comparison with our results.

We assume a uniform distribution with the range \([-a, a]\) where 0 is the center of the screen. We further assume subjects use a Gaussian likelihood to estimate the location of the stimulus. The resulting posterior distribution for the cue location becomes

\[
p(s | c) = \frac{\sqrt{\frac{2}{\pi}} e^{-\frac{(s-c)^2}{2\sigma^2}}}{\sigma \left( \text{erf}\left(\frac{a-c}{\sqrt{2}\sigma}\right) + \text{erf}\left(\frac{a+c}{\sqrt{2}\sigma}\right) \right)}, \quad \text{(posterior with uniform prior)} \tag{1}
\]
where $c$ is the cue location, $s$ is the stimulus, and $\sigma$ is the sensory noise.

Unfortunately, with this distribution we cannot obtain a closed form solution for the relationship between prior uncertainty and the JND. We numerically marginalize over the cue location for varying values of the stimulus. This results in a synthetic psychometric curve, representing the probability of a response for a given distance between stimuli. From this curve we obtain the JND. Repeating this procedure for various distributions we find the relationship between JND and prior uncertainty. We find that this relationship is nearly identical to that found when using a Gaussian prior (Fig. 1).

2 An alternative model for decision-making for the sampling hypothesis

There are multiple possibilities for how humans perform sampling. One possibility is that subjects first compute the posterior probability that one stimulus is greater than the other, and then sample from this distribution to make a choice; perhaps with a “majority-vote” mechanism. We examine this popular alternative model of sampling for any differences from our main predictions.

Mathematically, this model proposes that subjects first compute the probability that stimulus
\( s_2 \) is greater than \( s_1 \) based on the observed discrepancy between cues and their prior belief

\[
P(s_2 > s_1 | \delta_{\text{cue}}) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\delta_{\text{cue}}}{\sigma_s \sqrt{\sigma^2 + \sigma_s^2}} \right) \right].
\]  

(2)

Then subjects sample \( k \) times using this (Bernoulli) probability and pick the stimulus that appears more than \( k/2 \) times. The number of times a stimulus 2 is chosen follows a binomial distribution with probability given by Eq. 2. Therefore, the probability that the decision variable \( z = 1 \) is defined by

\[
P(z = 1 | \delta_{\text{cue}}) = I_p \left( \left\lfloor \frac{k}{2} \right\rfloor + 1, k - \left\lfloor \frac{k}{2} \right\rfloor \right),
\]  

(3)

where \( I_p \) is a regularized incomplete beta function, \( p = p(s_2 > s_1 | \delta_{\text{cue}}) \) and \( \lfloor \cdot \rfloor \) is the floor function. When \( k \) is even each stimulus may be sampled an equal number of times, allowing for a possible tie. However, through the definition of equation 3, ties are broken by choosing stimulus 1. While ties could be broken through other choices, our results indicate ties have an inconsequential effect on the overall results (Fig. 2). Following a similar numerical procedure as outlined in the previous section, we marginalized over the cues to predict how the JND changed with the prior uncertainty. We find that this model produced nearly indistinguishable predictions from our manuscript’s main result (Fig. 2).

3 Assessing possible key depress biases

There is the possibility of another bias: the key depressed by the subject. Subjects may be more likely to depress one key instead of the other, or they may display a bias for one key over the other when the discrepancy between cues is low (and the choice between them uncertain). We tested both types of biases simultaneously. Using a probit model within each subject and condition (narrow and wide prior), we assessed the effect of the key depressed, and the interaction between the absolute discrepancy and key depressed on the subject’s response. We found that neither effect was significant (\( p < 0.05 \), after Bonferroni correction, see Fig. 3).
Figure 2: Model predictions for the sampling model used in our article, the alternative sampling model, and MAP. In our sampling model subjects sample the position of each coin from its posterior distribution, and directly compare those estimates. The alternative sampling model computes the posterior over the 2AFC decision variable and samples from it to make a decision. The predictions from both sampling mechanisms are nearly identical.

Figure 3: We assessed whether the subjects biased their responses through key choice or through the interaction between key choice and stimuli discrepancy (distance between coins). Within subject and condition (narrow and wide priors), we assessed the effects’ significances with a probit regression. No significant effects were found (Bonferroni-corrected level $p$ value, depicted with the black line, $0.05/14 = 0.0035$).